

AD-A168 752

RESEARCH IN ADAPTIVE AND DECENTRALIZED STOCHASTIC  
CONTROL(U) TEXAS UNIV AT AUSTIN DEPT OF ELECTRICAL AND  
COMPUTER ENGINEERING S I MARCUS JAN 86

1/1

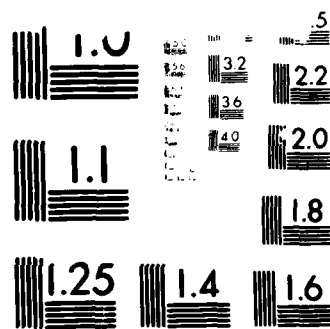
UNCLASSIFIED

AFOSR-TR-86-0295 AFOSR-84-0089

F/G 12/1

NL





Microcopy Resolution Test Chart

AD-A168 752

2

## REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S)			5. MONITORING ORGANIZATION REPORT NUMBER(S) <b>AFOSR-TR- 86 - 0295</b>		
6a. NAME OF PERFORMING ORGANIZATION University of Texas at Austin		6b. OFFICE SYMBOL (If applicable)		7a. NAME OF MONITORING ORGANIZATION AFOSR/NM	
6c. ADDRESS (City, State and ZIP Code) Dept. of Electrical & Computer Engineering Austin, TX 78712			7b. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR		8b. OFFICE SYMBOL (If applicable) NM		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-84-0089	
8c. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448			10. SOURCE OF FUNDING NOS.		
			PROGRAM ELEMENT NO. 61102F	PROJECT NO. 2304	TASK NO. A1
11. TITLE (Include Security Classification) <b>Research in Adaptive and Decentralized Stochastic Control (UNCLASSIFIED)</b>					
12. PERSONAL AUTHOR(S) Marcus, Steven I.					
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM 15 Mar 84 TO 14 Nov 85		14. DATE OF REPORT (Yr., Mo., Day) 1986 January	
				15. PAGE COUNT 8	
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)  Stochastic Adaptive Control, Decentralized Stochastic Control		
FIELD	GROUP	SUB. GR.			
19. ABSTRACT (Continue on reverse if necessary and identify by block number) Significant progress was made in a number of aspects of stochastic systems. The problem of adaptive control of priority assignment in queueing systems was solved. A distance-measures approach to the problem of approximation and identification of queueing systems was studied. A problem of adaptively controlling a discounted-reward finite-state Markov decision process was solved. Major new results were obtained for the problem of adaptive control with incomplete observations. In particular, we have studied in depth a problem of adaptive control with incomplete observations, in which the state is a finite state Markov process. In addition, earlier work on asymptotic approximations in nonlinear filtering was completed.					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS <input type="checkbox"/>			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a. NAME OF RESPONSIBLE INDIVIDUAL 86 6 10 002 Dr. Marc Jacobs			22b. TELEPHONE NUMBER (Include Area Code) (202) 767-4940		22c. OFFICE SYMBOL NM

Final Report

March 15, 1984 - November 14, 1985

Grant AFOSR-84-0089

"Research in Adaptive and Decentralized  
Stochastic Control"

Steven I. Marcus  
Department of Electrical and Computer Engineering  
University of Texas at Austin  
Austin, Texas 78712

January 6, 1986

A-1



Abstract

Significant progress was made in a number of aspects of stochastic systems. The problem of adaptive control of priority assignment in queueing systems was solved. A distance-measures approach to the problem of approximation and identification of queueing systems was studied. A problem of adaptively controlling a discounted-reward finite-state Markov decision process was solved. Major new results were obtained for the problem of adaptive control with incomplete observations. In particular, we have studied in depth a problem of adaptive control with incomplete observations, in which the state is a finite state Markov process. In addition, earlier work on asymptotic approximations in nonlinear filtering and on nonlinear systems with symmetries was completed, and new research on nonlinear discrete-time systems was initiated.

Approved for public release;  
distribution unlimited.

## I. SUMMARY OF RESEARCH PROGRESS AND RESULTS

During the twenty months of research supported by this grant, significant progress has been made in a number of aspects of stochastic systems. In this section this progress is summarized, and reference is made to the resulting publications listed in Section II.

### A. Adaptive Stochastic Control with Complete Observations

Our work on the adaptive stochastic control of queues with complete observations has continued. In [1], the priority assignment (or dynamic scheduling problem) in a queueing system with unknown arrival and service rate is considered. The long term average cost criterion with linear cost rates is solved, and the optimality of our proposed adaptive control algorithm is shown. A distance-measures approach to the problems of identification and approximation of queueing systems is presented in [2]; this approach combines ideas from statistical robustness, information-type measures, and parameter-continuity of stochastic processes.

In [3], we have considered general discounted-reward finite state Markov decision processes which depend on unknown parameters. An adaptive policy inspired by the nonstationary value iteration (NVI) scheme of Federgruen and Schweitzer is proposed; this is a variant of the usual method of successive approximations. It is shown that this adaptive policy is asymptotically discount optimal. This NVI policy is compared with the certainty equivalent or naive feedback control (NFC) policy. The NFC requires computation and storage of the optimal policy for all values of the parameter  $\theta$ ; this represents considerable off-line computation and considerable storage, particularly if the parameter set is not finite. On the other hand, the NVI policy requires

more on-line computation.

## B. Adaptive Stochastic Control with Incomplete Observations

The stochastic adaptive control problems solved to date had all, with the exception of those for linear systems, involved the assumption of complete (noiseless) state observations. As we proposed, we have begun a major new direction of research involving adaptive estimation and control problems for stochastic systems with incomplete (or noisy) observations of the state. We have already been successful in obtaining some important new results; the first of these are reported in [4]. In [4], we consider discounted Markov decision processes with incomplete state information and depending on unknown parameters. The process is first transformed into a completely observed Markov decision process and then (i) we use conditional least squares estimation to obtain a strongly consistent parameter estimation scheme, and this is combined with a nonstationary value-iteration procedure to obtain (ii) approximations converging uniformly to the optimal reward function, and (iii) asymptotically optimal adaptive policies.

This paper provides important general results, but the specific structure of the parameter estimation scheme must be studied in more detail. For this reason, we have begun an in-depth investigation into problems in which the state is a discrete time finite state Markov process [8]. In particular, the state is a finite state Markov chain  $x_t \in \{y_1, \dots, y_n\}$  with primitive transition matrix  $Q$ . The observation process  $y_t \in \{0, 1\}$ . If  $Q$  is known, there is a finite dimensional recursive filter for the conditional probability vector  $p_{t+1|t} = [p_{t+1|t}^1, \dots, p_{t+1|t}^n]^T$ , where  $p_{t+1|t} = P[x_{t+1}=y_i | y_0, \dots, y_t]$ .

In general, the adaptive estimation problem involves the computation of estimates (e.g., state estimates) in the presence of unknown parameters; in addition, estimates of the parameters are often computed simultaneously. In the present context, the adaptive estimation problem is that of computing recursive estimates of the conditional probability vector when the transition matrix  $Q$  is not completely known (i.e., it depends on a vector of unknown parameters  $\theta$ ). The approach to this problem which we investigate in [8] has been widely used in linear filtering: we use the previously derived recursive filter for the conditional probabilities, and we simultaneously recursively estimate the parameters, plugging the parameter estimates into the filter to obtain an approximate conditional probability vector  $\bar{p}_t|t-1$ . The recursive parameter estimator has the form

$$e_t = y_t - \gamma^T \bar{p}_t|t-1 \quad (1)$$

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \alpha_t R_t^{-1} \psi_t e_t \quad (2)$$

where  $\gamma = [\gamma_1, \dots, \gamma_n]^T$ ,  $\{\alpha_t\}$  is a sequence of positive scalars,  $R_t$  is a positive definite matrix which modifies the search direction, and  $-\psi_t$  is an approximation of the gradient of  $e_t$  with respect to  $\theta$  (evaluated at  $\hat{\theta}_{t-1}$ ).

The recursive stochastic algorithm we have described is of the general form

$$\eta_{k+1} = \eta_k + a_k G(\eta_k, \epsilon_k). \quad (3)$$

We follow the approach of Kushner to the Ordinary Differential Equation (ODE) Method of analyzing (3). That is, we define  $t_k = \sum_{i=1}^{k-1} (1/i)$  and  $m(t) = \max(k: t_k \leq t)$ ; thus  $m(t_k) = k$ . Let  $\bar{\eta}^0(\cdot)$  denote the piecewise linear interpolation of the function with value  $\eta_k$  at  $t_k$ . Define the

shifted function  $\bar{\eta}^k(\cdot)$  by  $\bar{\eta}^k(t) = \bar{\eta}^0(t+t_k)$ ,  $t \geq 0$ . Thus  $\bar{\eta}^k(0) = \eta_k$ , and the idea is to show weak convergence as  $k \rightarrow \infty$  of the sequence  $\{\bar{\eta}^k(\cdot)\}$  to the solution of an ODE, which can then be used to conclude properties (such as convergence as  $t \rightarrow \infty$ ) of the parameter estimates  $\hat{\theta}_t$ . The essential assumption is that  $\{\xi_k\}$  depends on  $\{\eta_k\}$  in such a way that if  $\eta_k = \eta$ , a constant, then  $\{\xi_k\}$  has a unique invariant (or stationary) measure. In our problem,  $\xi_k$  is a Markov process, and most of the paper [8] is devoted to an analysis that establishes that it has a unique invariant measure for fixed  $\eta$  (i.e., for fixed  $\theta$ ). This work represents the first major investigation into nonlinear adaptive stochastic control with incomplete observations.

### C. Other Research

We have completed work in other areas which were initially supported under Grant AFOSR-79-0025. In [7], asymptotic approximations for some nonlinear filtering problems were derived, analyzed, and compared with other filters. Lie algebraic and analytical methods were utilized; of particular interest was the estimation problem for linear systems with infrequently jumping parameters. In [5] and [6], the structure of nonlinear control systems with symmetries was studied, and the results obtained were used to derive reduced-order algorithms for certain classes of optimal control problems.

In addition, we have initiated research on the control of nonlinear discrete-time systems with the paper [9], in which we derive necessary and sufficient conditions for the approximate and local linearizability of such systems.



## II. PUBLICATIONS

### A. Journal Articles

- [1] O. Hernandez-Lerma and S.I. Marcus, "Optimal Adaptive Control of Priority Assignment in Queueing Systems," *Systems and Control Letters*, Vol. 4, April 1984, pp. 65-72.
- [2] O. Hernandez-Lerma and S.I. Marcus, "Identification and Approximation of Queueing Systems," *IEEE Transactions on Automatic Control*, Vol. AC-29, May 1984, pp. 472-474.
- [3] O. Hernandez-Lerma and S.I. Marcus, "Adaptive Control of Discounted Markov Decision Chains," *Journal of Optimization Theory and Applications*, Vol. 46, June 1985, pp. 227-235.
- [4] O. Hernandez-Lerma and S.I. Marcus, "Adaptive Control of Markov Processes with Incomplete State Information and Unknown Parameters," to appear in *Journal of Optimization Theory and Applications*.
- [5] J.W. Grizzle and S.I. Marcus, "Optimal Control of Systems Possessing Symmetries," *IEEE Transactions on Automatic Control*, Vol. AC-29, November 1984, pp. 1037-1040.
- [6] J.W. Grizzle and S.I. Marcus, "The Structure of Nonlinear Control Systems Possessing Symmetries," *IEEE Transactions on Automatic Control*, Vol. AC-30, March 1985, pp. 248-258.

### B. Other Publications

- [7] S.I. Marcus and E.K. Westwood, "On Asymptotic Approximations for Some Nonlinear Filtering Problems," *Proc. IFAC Triennial Congress*, Budapest, Hungary, July 2-6, 1984, Vol. VII, pp. 36-41.
- [8] S.I. Marcus and A. Arapostathis, "Analysis of an Identification Algorithm Arising in the Adaptive Estimation of Markov Chains," to appear in *Proc. 24th IEEE Conference on Decision and Control*, Ft. Lauderdale, Florida, December 10-13, 1985.
- [9] H.-G. Lee and S.I. Marcus, "Approximate and Local Linearizability of Nonlinear Discrete-Time Systems," submitted to the 1986 American Control Conference.

### III. PROFESSIONAL PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT

#### A. Partially Supported by this Grant

1. Steven I. Marcus, Principal Investigator
2. Hangju Cho, Research Assistant
3. Hong G. Lee, Research Assistant
4. Ian Walker, Research Assistant
5. Chang-Huan Liu, Postdoctoral Research Associate
6. Onesimo Hernandez-Lerma, Postdoctoral Research Associate
7. Aristotle Arapostathis, Associate Investigator

#### B. Not Supported by this Grant, but Contributors to the Research Effort

1. Jessy Grizzle, Ph.D. Candidate
2. Evan Westwood, M.S. Candidate

#### IV. PAPERS PRESENTED

1. S.I. Marcus, "Recent Developments in Nonlinear Estimation Theory," Distinguished Lecturer Series, Department of Electrical Engineering, University of Houston, University Park, Houston, Texas, April 9, 1984.
2. S.I. Marcus, "Optimal Adaptive Control of Queueing Systems," 2nd Istanbul Workshop on Large Scale Systems, June 24-27, 1984, Istanbul, Turkey.
3. S.I. Marcus and E.K. Westwood, "On Asymptotic Approximations for Some Nonlinear Filtering Problems," IFAC Triennial Congress, July 2-6, 1984, Budapest, Hungary.
4. J.W. Grizzle and S.I. Marcus, "A Jacobi-Liouville Theorem for Hamiltonian Control Systems," 23rd IEEE Conference on Decision and Control, December 12-14, 1984, Las Vegas, Nevada.
5. S.I. Marcus, "Analysis of an Adaptive Estimation Algorithm Arising in the Adaptive Estimation of Markov Chains," Workshop on the Estimation and Control of Stochastic Systems, Department of Mathematics, Mexican Polytechnic Institute, Mexico City, Mexico, March 27, 1985.

END

Dtic

7-86